Eighty-five presents were announced as having been received since the last meeting, including, amongst others :—

The collected works of Arthur Cayley, vol. xiii., presented by the Cambridge University Press ; Annals of the Royal Observatory, Cape of Good Hope, vol. iii. (The Cape Photographic Durchmusterung, Part 1) ; vols. vi., vii. (Determination of the Solar Parallax and the Mass of the Moon from Heliometer Observations of *Iris*, *Victoria*, and *Sappho*) ; Results of Meridian Observations made during 1861–65 under the direction of Sir Thomas Maclear, reduced and printed under the direction of David Gill ; Appendix to Cape Meridian Observations 1890–91 (Star Correction Tables by W. H. Finlay), presented by the Observatory.

On the Parallax of Sirius and of α Gruis. By David Gill, C.B., F.R.S., &c., Her Majesty's Astronomer at the Cape.

In connection with the preparation for press of a work containing determinations of the parallaxes of the 1st magnitude stars of the Southern Hemisphere, I have recently discussed a series of observations for the parallax of *Sirius* made by me with the Cape Heliometer in the years 1888 and 1889.

The results seem to be of sufficient interest for communication in a brief form to the Society. The details will appear subsequently in the *Annals* of the Cape Observatory.

The selected comparison stars were :—

γ = B.D. − 16° 1593 ;	mag. 8·7 ;	P. angle from *Sirius*	279°·17 ;	S = 4310″
δ = B.D. − 16° 1623 ;	„ 8·7 ;	„ „ „	101·26 ;	S = 4536
		Difference ...	177°·91	226″
		Sum ...		8846″

The parallax factors for the difference and sum of the distances are :—

$$\text{For } \delta-\gamma \quad 1{\cdot}997\ \mathrm{R}\cos(\odot - 15^\circ{\cdot}6)$$
$$\delta+\gamma \quad 0{\cdot}023\ \mathrm{R}\cos(\odot - 105^\circ{\cdot}5)$$

so that the mean position angle is nearly in the major axis of the parallactic ellipse. The pair is thus a very favourable one for our purpose, except that the comparison stars are rather too faint for the most exact measurement.

The observations were arranged symmetrically—*i.e.* in the order γ, δ, δ, γ, or δ, γ, γ, δ—so that the mean instant of observation for each distance is nearly the same.

The light of *Sirius* was diminished by suitable wire gauze screens till the reduced image was similar in every respect to the images of the comparison stars when the latter were viewed through the non-obscured segment of the object glass. The

reversing prism was used symmetrically throughout, so as entirely to eliminate any subjective error depending on the direction of measurement with respect to the vertical.

The heliometer scale readings, converted into arc with an adopted mean scale-value, were corrected for errors of the screw and scale divisions, for refraction, aberration, and proper motion to 1889·0.

The sum and difference of the two mean measured distances γ and δ were then taken for each night of observation, and the differences were then corrected for variation of scale-value on the assumption that the sum of the distances should be a constant, the correction being

$$0{\cdot}025 \quad \{8846''{\cdot}62-(\gamma+\delta)\}.$$

In only three instances did this scale-value correction amount to $0''{\cdot}01$.

Putting $(\delta'-\gamma')$ for the values of $(\delta-\gamma)$ thus corrected, and assuming the true mean value of

$$\delta'-\gamma'=226''{\cdot}100+x,$$

and adopting for the proper motion of the centre of gravity of the system of *Sirius*

$$\mu_\alpha=-0^s{\cdot}0374\,(t-1889{\cdot}0)$$
$$\mu_\delta=-1''{\cdot}2064\,(t-1889{\cdot}0)$$

and taking the orbital motion of the bright component of the system from Auwers' table, *Ast. Nach.* 3085, the following equations of condition were derived :—

		Weight.	O–C.
1888 Mar. 30	$x-0{\cdot}76y+1{\cdot}97\pi=+0''{\cdot}506-1{\cdot}52\Delta\beta$	1	−0·018
Apr. 2	$x-\ {\cdot}75\ +1{\cdot}99\ =+\ {\cdot}523-1{\cdot}76$	1	− ·009
23	$x-\ {\cdot}69\ +1{\cdot}91\ =+\ {\cdot}598-2{\cdot}06$	1	+ ·092
26	$x-\ {\cdot}68\ +1{\cdot}88\ =+\ {\cdot}500-1{\cdot}56$	1	+ ·004
27	$x-\ {\cdot}68\ +1{\cdot}87\ =+\ {\cdot}431-1{\cdot}94$	1	− ·061
Sept. 28	$x-\ {\cdot}26\ -1{\cdot}97\ =-\ {\cdot}821+1{\cdot}20$	1	+ ·096
29	$x-\ {\cdot}26\ -1{\cdot}97\ =-\ {\cdot}817+1{\cdot}10$	½	+ ·090
Oct. 1	$x-\ {\cdot}25\ -1{\cdot}98\ =-\ {\cdot}916+1{\cdot}42$	1	− ·006
2	$x-\ {\cdot}25\ -1{\cdot}99\ =-1{\cdot}019+1{\cdot}27$	1	− ·106
4	$x-\ {\cdot}24\ -1{\cdot}99\ =-\ {\cdot}905+1{\cdot}37$	1	+ ·008
7	$x-\ {\cdot}23\ -1{\cdot}99\ =-\ {\cdot}950+1{\cdot}13$	1	− ·038
1889 Mar. 24	$x+\ {\cdot}22\ +1{\cdot}95\ =+\ {\cdot}537-2{\cdot}38$	½	− ·033
Apr. 21	$x+\ {\cdot}30\ +1{\cdot}93\ =+\ {\cdot}364-2{\cdot}40$	1	− ·202
22	$x+\ {\cdot}30\ +1{\cdot}92\ =+\ {\cdot}849-1{\cdot}59$	½	+ ·287
23	$x+\ {\cdot}31\ +1{\cdot}91\ =+\ {\cdot}465-1{\cdot}50$	½	− ·095
25	$x+\ {\cdot}31\ +1{\cdot}89\ =+\ {\cdot}674-1{\cdot}63$	1	+ ·122

On 1888 September 29 and 1889 March 24, April 22 and 23 the images were very diffused (I 3, S 3 and 3–4), and the corresponding equations have received half weight.

In these equations

y is the correction applicable to the adopted correction for the annual change of $(\delta' - \gamma')$.

π is the parallax of *Sirius*.

$\Delta\beta$ is the correction to the constant of refraction of the principal star as compared with that of the comparison star. That is to say, if the refraction of the comparison stars in zenith distance is

$$\beta \tan \zeta,$$

that of the principal star is assumed to be

$$(\beta + \Delta\beta) \tan \zeta,$$

and the factor of $\Delta\beta$ is $\tan \zeta \cos (p - q)$.

Since the observations were not so arranged as to permit the determination of $\Delta\beta$, we find the values of x, y, and π in terms of $\Delta\beta$.

Having regard to the weights, the normal equations are :—

$$\begin{array}{rrrrr}
+14{\cdot}00x & -3{\cdot}90y & +\ 5{\cdot}43\pi & = -\ 0''{\cdot}498 & -\ 9{\cdot}38\Delta\beta \\
-\ 3{\cdot}90 & +3{\cdot}18 & -\ 2{\cdot}20 & -\ 0{\cdot}007 & +\ 3{\cdot}13 \\
+\ 5{\cdot}43 & -2{\cdot}20 & +53{\cdot}01 & +18{\cdot}640 & -45{\cdot}15
\end{array}$$

Solving these, and substituting the resulting values of x, y, and π in the original equations we get the following results :—

$$\begin{array}{llll}
x = -0''{\cdot}164 - 0{\cdot}36\Delta\beta; & \text{weight } 7{\cdot}4; & \text{probable error} & \pm 0''{\cdot}0257 \\
y = +0{\cdot}053 - 0{\cdot}02\Delta\beta; & \quad ,,\quad 2{\cdot}1; & \quad ,, \quad ,, & \pm 0{\cdot}0485 \\
\pi = +0{\cdot}370 - 0{\cdot}82\Delta\beta; & \quad ,,\quad 50{\cdot}7; & \quad ,, \quad ,, & \pm 0{\cdot}0097
\end{array}$$

$$[pvv] = 0{\cdot}1405 \qquad \frac{[pvv]}{n-3} = 0{\cdot}0108$$

and the probable error of an observation of weight unity $\pm 0''{\cdot}070$. The residuals are given in the right-hand column beside the equations.

It is very improbable that the value of $\Delta\beta$ is a sensible quantity. In the years 1881–83 I made a series of seventy-nine observations with my 4-inch heliometer (*Mem. R.A.S.* vol. xlviii. pp. 83–97) to determine the parallax of *Sirius* relative to the stars, Lal. 12936, mag. $7\frac{3}{4}$, and Lal. 13129, mag. 8, and found for the value of the parallax

$$\pi = +0''{\cdot}370; \text{ probable error } \pm 0''{\cdot}009.$$

In this series the systematic difference between E. and W. observations was only 0″·007, and the effect of regarding x_W as different from x_E was to change the resulting value of the parallax by only 0″·001 from that found by the assumption that none of the observations were affected by systematic error. The exact agreement of the present result with that of my previous determination—notwithstanding a difference of about 40° in position angle of the comparison stars—points also to non-existence of systematic error depending on $\Delta\beta$.

We have now from reliable heliometer observations the following results for the parallax of *Sirius* :—

			Prob. Error.	Mean. Prob. Error.
Gill	Cape	1881–83	+0″·370 ± 0″·009 *	+0″·370 ± 0″·007
	Cape	1888–89	+0·370 ± 0·0097 †	
Elkin...	Cape	1881–82	+0″·407 ± 0″·018 ‡	+0″·407 ± 0″·018
			Mean	+0″·374 ± 0″·006

We may therefore regard the parallax of *Sirius* relative to average comparison stars of about $8\frac{1}{2}$ magnitude as now satisfactorily determined, and the corrections depending on a parallax of 0″·37 may with advantage be introduced in the apparent places of *Sirius* given in the national ephemerides.

Parallax of α Gruis.

It is stated above that the comparison stars employed for *Sirius* are rather too faint for the most exact measurement. In the case of α *Gruis* I found a very symmetrical pair of comparison stars of 8th magnitude, and as the observations yield the smallest residuals which I have yet met with, the results of the observations may be of interest as illustrating the accuracy attainable with a good heliometer under the most favourable conditions.

The comparison stars were

γ = G. xxi. 1712	mag. 8.	P. angle from α *Gruis*	259°·6 ;	distance 3670″
δ = G. xxii. 187	„ 8.	„ „ „	77·8 ;	„ 3510
		Difference	181°·8	160″

with the parallax factors

$$\text{For } \gamma-\delta \quad 1{\cdot}963 \text{ R} \cos(\odot - 51^\circ{\cdot}5)$$

$$\gamma+\delta \quad 0{\cdot}018 \text{ R} \cos(\odot - 22^\circ{\cdot}6)$$

* *Mem. R.A.S.* vol. xlviii. pp. 83–97. † Present paper.
‡ *Mem. R.A.S.* vol. xlviii. pp. 97–116.

The observations were made with the same precautions as in the case of *Sirius*. Kobold's value of the proper motion of *a Gruis* (*Ast. Nach.* 3435), based on Auwers' researches, was adopted, viz.—

$$0''{\cdot}205 \text{ in the direction } p\ 148^\circ{\cdot}8.$$

Projecting the correction for this motion on the observed distances, and forming equations of condition in the same manner as those for *Sirius*, we get :—

1888		Weight.	O–C.
Apr. 27	$x-0{\cdot}68y+1{\cdot}92\pi=+0''{\cdot}037-1{\cdot}71\Delta\beta$	1	$+0''{\cdot}032$
May 1	$x-{\cdot}67+1{\cdot}95=-{\cdot}082-1{\cdot}88$	1	$-{\cdot}087$
3	$x-{\cdot}66+1{\cdot}96=+{\cdot}119-1{\cdot}75$	1	$+{\cdot}112$
8	$x-{\cdot}65+1{\cdot}98=-{\cdot}049-1{\cdot}38$	1	$-{\cdot}056$
Nov. 19	$x-{\cdot}12-1{\cdot}93=-{\cdot}016+2{\cdot}08$	1	$+{\cdot}015$
22	$x-{\cdot}11-1{\cdot}91=+{\cdot}043+1{\cdot}58$	½	$+{\cdot}073$
27	$x-{\cdot}10-1{\cdot}88=-{\cdot}025+2{\cdot}05$	½	$+{\cdot}004$
29	$x-{\cdot}09-1{\cdot}86=+{\cdot}010+1{\cdot}77$	½	$+{\cdot}039$
Dec. 8	$x-{\cdot}06-1{\cdot}75=-{\cdot}103+1{\cdot}98$	1	$-{\cdot}078$
1889			
May 9	$x+{\cdot}35+1{\cdot}98=+{\cdot}025-1{\cdot}56$	1	$-{\cdot}022$
31	$x+{\cdot}41+1{\cdot}89=+{\cdot}067-1{\cdot}66$	1	$+{\cdot}019$
June 3	$x+{\cdot}42+1{\cdot}85=+{\cdot}054-1{\cdot}58$	1	$+{\cdot}006$

On November 22, 27, and 29 the observations were made with definition 3, and the corresponding equations have received half weight.

Having regard to the weights, the normal equations are :—

$$\begin{aligned}
&+10{\cdot}50x-1{\cdot}81y+7{\cdot}03\pi=+0''{\cdot}066-4{\cdot}76\Delta\beta\\
&-\ \ 1{\cdot}81\ \ +2{\cdot}26\ \ -\ 2{\cdot}33\ =+0{\cdot}048+\ 1{\cdot}95\\
&+\ \ 7{\cdot}03\ \ -2{\cdot}33\ \ +38{\cdot}27\ =+0{\cdot}507-34{\cdot}82
\end{aligned}$$

Solving these normals and substituting the values of x, y, and π, we get the residuals given in the column headed O—C. The results are :—

$$\begin{aligned}
x&=+0''{\cdot}003+0{\cdot}18\Delta\beta\ ;\ \text{weight } 9{\cdot}1\ ;\ \text{probable error } \pm 0''{\cdot}014\\
y&=+0{\cdot}040+0{\cdot}04\ \ ;\ \ ,,\ \ 1{\cdot}9\ ;\ \ ,,\ \ ,,\ \ \pm 0{\cdot}030\\
\pi&=+0{\cdot}015-0{\cdot}94\ \ ;\ \ ,,\ \ 32{\cdot}9\ ;\ \ ,,\ \ ,,\ \ \pm 0{\cdot}007
\end{aligned}$$

$$[pvv]=0{\cdot}0348 \qquad \frac{[pvv]}{n-3}=0{\cdot}00386$$

and the probable error of a single observation of weight unity $\pm 0''{\cdot}042$.

The only remaining uncertainty in this result is the possible refraction effect, $\Delta\beta$, depending on the possible difference between the mean refrangibility of the light of *α Gruis* and that of the comparison stars.

As this effect, if at all sensible, must certainly depend on the difference between the types of the spectra of the principal star and the comparison star, it seems better to defer a discussion of the matter till the data referred to in a paper which I recently communicated to the Society (*Monthly Notices*, vol. lviii. p. 53) have been collected, and until I have access to the McClean telescope. The whole of this question, as it affects all these parallax researches, can then be dealt with on a broad and satisfactory basis. Meanwhile I venture to express the belief, founded on other considerations, that the value of $\Delta\beta$ will in this case not be found to amount to $0''\cdot01$.

It may interest the more casual reader to point out the great accuracy of the individual observations which the preceding results imply.

The probable error of the single observation of difference of two opposite distances is thus :—

For *Sirius* and comparison stars of	$8\frac{3}{4}$ mag.	$\pm 0''\cdot070$
„ *α Gruis* „ „	8·0 mag.	$\pm 0\cdot042$

This implies that, as the result of one complete set of observations lasting less than an hour, the position of the principal star may be projected on the great circle joining two comparison stars with a probable error of

$\pm 0''\cdot035$ in the case of stars $8\frac{3}{4}$ mag.

or

$\pm 0\cdot021$ in the case of stars 8·0 mag.

a precision which, so far as I am aware, has not been approached by any other method of observation, nor previously attained in astronomical researches of any kind. It becomes thus possible to detect any differential stellar parallax amounting to $0''\cdot02$ with perfect certainty by a comparatively small number of observations.

The Double Star ζ Boötis. Σ 1865. By S. W. Burnham.

This double star was discovered by Herschel a little more than one hundred years ago (=H. N. 114). In his first observation, 1796 April 5, it is described as "very nearly in contact; I can, however, see a small division." On the following evening he measured the position-angle, which is given as 41′ 59″ *n.p.*